

# Chapter 1

## Easy Problems

1.1.  $y = \tan x$

1.2.  $f(x) = g(x) \ln(g(x))$ .

1.3.  $y = \arctan x = \tan^{-1} x$

1.4.  $y = \arcsin(x)$

1.5.  $y = (x + 1) \ln(x + 1)$ .

## Chapter 2

# Probability Spaces

- 2.1. A coin is weighted so that heads is four times as likely as tails. Find the probability that: (a) tails appears, (b) heads appears
- 2.2. Under which of the following functions does  $S = \{a_1, a_2\}$  become a probability space?
- (a)  $P(a_1) = \frac{1}{3}, P(a_2) = \frac{1}{2}$     (b)  $P(a_1) = \frac{3}{4}, P(a_2) = \frac{1}{4}$   
(c)  $P(a_1) = 1, P(a_2) = 0$     (d)  $P(a_1) = \frac{5}{4}, P(a_2) = -\frac{1}{4}$

# Appendix A

## Solutions

1.1

$$\begin{aligned}y &= \tan x \\&= \frac{\sin x}{\cos x} \\ \frac{dy}{dx} &= \frac{\cos x}{\cos x} + \sin x \times \frac{-1}{\cos^2 x} \times -\sin x \\&= 1 + \tan^2 x \\&= \sec^2 x.\end{aligned}$$

1.2

$$\begin{aligned}f'(x) &= g'(x) \ln(g(x)) + \frac{g(x)}{g(x)} g'(x) \\&= g'(x)(1 + \ln(g(x))).\end{aligned}$$

1.3

$$\tan y = x$$

diff w.r.t.  $x$ :

$$\begin{aligned}\sec^2 y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\sec^2 y} \\&= \frac{1}{1 + \tan^2 y} \\&= \frac{1}{1 + x^2}\end{aligned}$$

1.4

$$\sin(y) = x$$

diff. w.r.t.  $x$ :

$$\begin{aligned}\cos y \frac{dy}{dx} &= 1 \\ \frac{dy}{dx} &= \frac{1}{\cos y} \\ &= \frac{1}{\sqrt{1 - \sin^2 y}} \\ &= \frac{1}{\sqrt{1 - x^2}}.\end{aligned}$$

1.5

$$\begin{aligned}\frac{dy}{dx} &= \ln(x+1) + \frac{x+1}{x+1} \\ &= 1 + \ln(x+1).\end{aligned}$$

2.1 Let  $p = P(T)$ , then  $P(H) = 4p$ . We require  $P(H) + P(T) = 1$ , so  $4p + p = 1$ , hence  $p = \frac{1}{5}$ . Therefore: i.  $P(T) = \frac{1}{5}$ , ii.  $P(H) = \frac{4}{5}$

2.2 2.2b and 2.2c